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# A Four-site Higgsless Model with Wavefunction Mixing

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Motivated by models of holographic technicolor, we discuss a four-site deconstructed Higgsless model with nontrivial wavefunction mixing. We compute the spectrum of the model, the electroweak triple gauge boson vertices, and, for brane-localized fermions, the electroweak parameters to  $\mathcal{O}(M_W^2/M_\rho^2)$ . We discuss the conditions under which  $\alpha S$  vanishes (even for brane-localized fermions) and the (distinct but overlapping) conditions under which the phenomenologically interesting decay  $a_1 \rightarrow W\gamma$  is non-zero and suppressed by only one power of  $(M_W/M_\rho)$ .

## I. INTRODUCTION

Higgsless models of electroweak symmetry breaking [1] may be viewed as “dual” to more conventional technicolor models [2, 3] and, as such, provide a basis for constructing low-energy effective theories to investigate the phenomenology of a strongly interacting symmetry breaking sector [4, 5]. One approach to constructing such an effective theory, the three-site model [6], includes only the lightest of the extra vector mesons typically present in such theories – the meson analogous to the  $\rho$  in QCD. An alternative approach is given by “holographic technicolor” [7], which potentially provides a description of the first two extra vector mesons – including, in addition to the  $\rho$ , the analog of the  $a_1$  meson in QCD.

In this note we consider a four-site “Higgsless” model [8] illustrated, using “moose notation” [9], in fig. 1. We show how, once an  $L_{10}$ -like “wavefunction” mixing term for the two strongly-coupled  $SU(2)$  groups in the center of the moose is included, we can reproduce the features of the holographic model – including the vanishing of the parameter  $\alpha S$  for brane-localized fermions and the existence (whether or not  $\alpha S = 0$ ) of the potentially interesting decay  $a_1 \rightarrow W\gamma$ .

## II. THE MODEL

The Lagrangian for the model consists of several parts. First, the usual nonlinear sigma model link terms

$$\mathcal{L}_\pi = \frac{f_1^2}{4} \left[ \text{Tr} D^\mu \Sigma_1 D_\mu \Sigma_1^\dagger + \text{Tr} D^\mu \Sigma_3 D_\mu \Sigma_3^\dagger \right] + \frac{f_2^2}{4} \text{Tr} D^\mu \Sigma_2 D_\mu \Sigma_2^\dagger. \quad (1)$$

Next, the gauge-boson kinetic energies

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} \left( \vec{W}_{0\mu\nu}^2 + \vec{W}_{1\mu\nu}^2 + \vec{W}_{2\mu\nu}^2 + \vec{W}_{3\mu\nu}^2 \right), \quad (2)$$

where we denote the weakly-coupled  $SU(2) \times U(1)$  fields by  $\vec{W}_0$  and  $\vec{W}_3 \equiv B$  (by convention,  $i = 3$  vanishes for the charged sector), and the strongly coupled  $SU(2)$  fields by  $\vec{W}_{1,2}$ . And finally, there is an  $L_{10}$ -like mixing between the

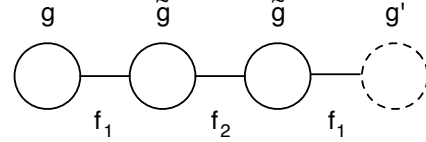


FIG. 1: The “moose” diagram [9] for the  $SU(2)^3 \times U(1)$  model considered in this note. The solid circles represent  $SU(2)$  groups; the dashed circle, a  $U(1)$  group; the “links”,  $SU(2) \times SU(2)/SU(2)$  non-linear sigma models. In order to be phenomenologically realistic [10], we work in the limit  $g, g' \ll \tilde{g}$ ; in this limit the model also has an approximate parity symmetry. We consider brane-localized fermions, which couple only the  $SU(2) \times U(1)$  at the ends of the moose, and add an  $L_{10}$ -like “wavefunction mixing” term to mix the two strongly-coupled  $SU(2)$  groups in the middle two sites.

middle two sites

$$\mathcal{L}_\varepsilon = -\frac{\varepsilon}{2} \text{Tr} \left[ \vec{W}_{1\mu\nu} \Sigma_2 \vec{W}_2^{\mu\nu} \Sigma_2^\dagger \right], \quad (3)$$

where in this calculation we treat  $\varepsilon$  as an  $\mathcal{O}(1)$  parameter. This model has a “parity” (more precisely, a  $G$ -parity) symmetry in the  $g = g' = 0$  limit, under which  $\vec{W}_1^\mu \rightarrow \vec{W}_2^\mu$ ,  $\Sigma_1 \rightarrow \Sigma_3^\dagger$ , and  $\Sigma_2 \rightarrow \Sigma_2^\dagger$ . In the limit  $f_2 \rightarrow \infty$ ,<sup>1</sup> this model reduces to the three-site model considered in [6].

In unitary gauge (with  $\Sigma_1 = \Sigma_2 = \Sigma_3 \equiv \mathcal{I}$ ), the  $\mathcal{L}_\varepsilon$  term above corresponds to wavefunction-mixing of the fields  $\vec{W}_i$ ,

$$\mathcal{L} = -\frac{1}{4} \vec{W}_{i\mu\nu} \tilde{Z}_{ij} \vec{W}_j^{\mu\nu} - \frac{1}{2} \vec{W}_{i\mu} M_{ij}^2 \vec{W}_j^\mu, \quad (4)$$

with

$$\tilde{Z} = \begin{pmatrix} 1 & & \\ & 1 & \varepsilon \\ & \varepsilon & 1 \\ & & & 1 \end{pmatrix}. \quad (5)$$

To avoid ghosts, we require  $\tilde{Z}$  to be positive-definite, and hence  $|\varepsilon| < 1$ .

<sup>1</sup> For fixed values of  $2/f_1^2 + 1/f_2^2$ , see eqn. (23).

### III. MASSES AND MIXING ANGLES

The eigenstates corresponding to the quadratic part of Lagrangian in eqn. (4) satisfy the generalized eigenvalue equation

$$M^2 \vec{v}_n = m_n^2 \tilde{Z} \vec{v}_n, \quad (6)$$

where  $\vec{v}_n$  is a vector in site-space with components  $v_n^i$ . The superscript  $i$  labels the sites, running from 0 to 2 for charged-bosons ( $n = W^\pm, \rho^\pm, a_1^\pm$ ), and 0 to 3 for neutral ones ( $n = Z^0, \rho^0, a_1^0, \gamma$ ). If we choose eigenvectors normalized by  $\vec{v}_n^T \tilde{Z} \vec{v}_m = \delta_{nm}$ , the gauge-eigenstate ( $W_\mu^i$ ) and mass-eigenstate ( $W'_{n\mu}$ ) fields are related by

$$W_\mu^i = \sum_n v_n^i W'_{n\mu}. \quad (7)$$

#### A. The $g = g' = 0$ Limit

Consider first the  $g = g' = 0$  limit, in which we can determine the leading contributions to the heavy gauge-boson masses. Due to the parity symmetry in this limit, we expect the eigenvectors to be proportional to  $\vec{W}_1^\mu \pm \vec{W}_2^\mu$ . Applying the normalization condition  $\vec{v}_n^T \tilde{Z} \vec{v}_m = \delta_{nm}$ , we find a parity-even eigenvector (the “ $\rho$ ”)

$$\vec{\rho}^\mu = \frac{1}{\sqrt{2(1+\varepsilon)}} \left( \vec{W}_1^\mu + \vec{W}_2^\mu \right), \quad (8)$$

with mass

$$m_\rho^2 = \frac{\tilde{g}^2}{4} \frac{f_1^2}{1+\varepsilon}, \quad (9)$$

and a parity-odd eigenvector (the “ $a_1$ ”)

$$\vec{a}_1^\mu = \frac{1}{\sqrt{2(1-\varepsilon)}} \left( \vec{W}_1^\mu - \vec{W}_2^\mu \right), \quad (10)$$

with mass

$$m_{a_1}^2 = \frac{\tilde{g}^2}{4} \frac{f_1^2 + 2f_2^2}{1-\varepsilon}. \quad (11)$$

We note that the  $\rho$  and  $a_1$  are degenerate for

$$\varepsilon = -\frac{f_2^2}{f_1^2 + f_2^2}, \quad (12)$$

a value satisfying the constraint  $|\varepsilon| < 1$ . As  $\varepsilon$  becomes more negative, the  $a_1$  becomes lighter than the  $\rho$ .

#### B. The Photon

Examining the eigenvalue eqn. (6) we see that the wavefunction factor  $\tilde{Z}$  affects the normalization of a

massless eigenvector, but not the orientation. We see, therefore, that the photon must be of the form

$$A_\mu = \frac{e}{g} W_{0\mu}^3 + \frac{e}{\tilde{g}} W_{1\mu}^3 + \frac{e}{\tilde{g}} W_{2\mu}^3 + \frac{e}{g'} B_\mu, \quad (13)$$

or

$$(v_\gamma)^T = \left( \frac{e}{g}, \frac{e}{\tilde{g}}, \frac{e}{\tilde{g}}, \frac{e}{g'} \right). \quad (14)$$

The electric charge  $e$  is, then, determined from the normalization condition to be

$$\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2} + \frac{2(1+\varepsilon)}{\tilde{g}^2}. \quad (15)$$

Examining the photon-couplings, we see that the unbroken gauge-generator has the expected form  $Q = T^3 + T_1^3 + T_2^3 + Y$ .

#### C. The $W$ -boson

Next, we consider a perturbative evaluation of the electroweak boson eigenvectors and eigenvalues, computed in powers of  $x = g/\tilde{g}$ . We start with the  $W$ -boson; the charged-boson mass matrix is given by

$$M_W^2 = \frac{\tilde{g}^2}{4} \begin{pmatrix} x^2 f_1^2 & -x f_1^2 & 0 \\ -x f_1^2 & f_1^2 + f_2^2 & -f_2^2 \\ 0 & -f_2^2 & f_1^2 + f_2^2 \end{pmatrix}. \quad (16)$$

To  $\mathcal{O}(x^2)$  we find

$$\begin{aligned} v_W^0 &= \left[ 1 - \frac{f_1^4 + 2(1+\varepsilon)f_1^2 f_2^2 + 2(1+\varepsilon)f_2^4}{2(f_1^2 + 2f_2^2)^2} x^2 \right], \\ v_W^1 &= x \frac{f_1^2 + f_2^2}{f_1^2 + 2f_2^2} W_1, \\ v_W^2 &= x \frac{f_2^2}{f_1^2 + 2f_2^2} W_2, \end{aligned} \quad (17)$$

where we have computed, but do not display, the corrections of  $\mathcal{O}(x^3)$  to the last two components. For the corresponding eigenvalue we find

$$m_W^2 = \frac{g^2}{4} \frac{f_1^2 f_2^2}{f_1^2 + 2f_2^2} \left[ 1 - \frac{f_1^4 + 2(1+\varepsilon)f_1^2 f_2^2 + 2(1+\varepsilon)f_2^4}{(f_1^2 + 2f_2^2)^2} x^2 \right]. \quad (18)$$

#### D. The $Z$ -boson

The neutral gauge-boson mass matrix is

$$M_Z^2 = \begin{pmatrix} x^2 f_1^2 & -x f_1^2 & 0 & 0 \\ -x f_1^2 & f_1^2 + f_2^2 & -f_2^2 & 0 \\ 0 & -f_2^2 & f_1^2 + f_2^2 & -x \tan \theta f_1^2 \\ 0 & 0 & -x \tan \theta f_1^2 & x^2 \tan^2 \theta f_1^2 \end{pmatrix}. \quad (19)$$

where we have defined the angle  $\theta$  by  $g'/g \equiv \tan \theta$ . Note that  $\theta$  is the *leading order* weak mixing angle; we will later define a weak mixing angle  $\theta_Z$  that is better suited to comparison with experiment. We have computed the  $Z$ -

boson eigenvector to  $\mathcal{O}(x^3)$  – as the result is complicated, and the algebra unilluminating, we do not reproduce it here. For the  $Z$ -boson mass, we find

$$m_Z^2 = \frac{g^2}{4 \cos^2 \theta} \frac{f_1^2 f_2^2}{f_1^2 + 2f_2^2} \left[ 1 - \frac{(3 - \varepsilon)f_1^4 + 4(1 + \varepsilon)(f_1^2 f_2^2 + f_2^4) + (1 + \varepsilon)(f_1^2 + 2f_2^2)^2 \cos 4\theta}{4(f_1^2 + 2f_2^2)^2} x^2 \sec^2 \theta \right]. \quad (20)$$

#### IV. THE ELECTROWEAK PARAMETERS

From eqn. (7), we can compute the couplings of the mass-eigenstate electroweak gauge-bosons to fermions. For brane-localized fermion couplings of the form

$$\mathcal{L}_f = g_0 \bar{J}_L^\mu \cdot \vec{W}_\mu^0 + g' J_Y^\mu B_\mu, \quad (21)$$

we find the mass-eigenstate  $W$ -boson couplings  $g_W^f = g_0 v_W^0$  and the  $Z$ -boson couplings

$$g_Z^f = g v_Z^0 I_3 + g' v_Z^3 Y = g I_3 (v_Z^0 - \tan \theta v_Z^3) + g' v_Z^3 Q. \quad (22)$$

We may then compute the on-shell precision electroweak parameters at tree-level to  $\mathcal{O}(x^2)$ , using the definitions and procedures outlined in [10, 11]. The values of electric charge, eqn. (15), and  $m_Z^2$ , eqn. (20), are given above, and we find the Fermi constant

$$\sqrt{2} G_F = \frac{1}{v^2} = \frac{2}{f_1^2} + \frac{1}{f_2^2}, \quad (23)$$

where  $v \approx 246$  GeV.

The only non-zero precision electroweak parameter parameter is  $\alpha S$  [12], for which we find

$$\frac{\alpha S}{4s^2} = \frac{\varepsilon f_1^4 + 2(1 + \varepsilon)f_1^2 f_2^2 + 2f_2^4(1 + \varepsilon)}{(f_1^2 + 2f_2^2)^2} x^2, \quad (24)$$

As expected [5, 7], we can choose  $\varepsilon$  so that  $\alpha S$  vanishes for any given value of  $f_1/f_2$

$$\varepsilon \rightarrow -\frac{2(f_2^4 + f_1^2 f_2^2)}{f_1^4 + 2f_1^2 f_2^2 + 2f_2^4}, \quad (25)$$

while satisfying  $|\varepsilon| < 1$ .

Note, however, that the value of the low-energy parameter  $|\varepsilon|$  that makes  $\alpha S$  vanish is of order one, larger than would be expected by naive dimensional analysis [13]. This result is consistent with investigations of continuum 5d effective theories [14, 15], and with investigations of plausible conformal technicolor “high-energy completions” of this model using Bethe-Salpeter methods [16, 17], both of which suggest that  $\alpha S > 0$  and that it may not be possible to achieve very small values of  $\alpha S$ . We note also that the result is consistent with the expectation of [18, 19], since the value of  $\varepsilon$  required for  $\alpha S$

to vanish results in axial-vector mesons which are lighter than the vector mesons.<sup>2</sup>

#### V. TRIPLE BOSON VERTICES

##### A. Electroweak Vertices

Consider the electroweak vertices  $\gamma WW$  and  $ZWW$ . To leading order, in the absence of CP-violation, the triple gauge boson vertices may be written [23]

$$\begin{aligned} \mathcal{L}_{TGV} = & -ie \frac{c_Z}{s_Z} [1 + \Delta \kappa_Z] W_\mu^+ W_\nu^- Z^{\mu\nu} \\ & - ie [1 + \Delta \kappa_\gamma] W_\mu^+ W_\nu^- A^{\mu\nu} \\ & - ie \frac{c_Z}{s_Z} [1 + \Delta g_1^Z] (W^{+\mu\nu} W_\mu^- - W^{-\mu\nu} W_\mu^+) Z_\nu \\ & - ie (W^{+\mu\nu} W_\mu^- - W^{-\mu\nu} W_\mu^+) A_\nu, \end{aligned} \quad (26)$$

where the two-index tensors denote the Lorentz field-strength tensor of the corresponding field. In the standard model,  $\Delta \kappa_Z = \Delta \kappa_\gamma = \Delta g_1^Z \equiv 0$ . Note that the expressions for  $\kappa_Z$  and  $g_1^Z$  involve  $c_Z \equiv \cos \theta_Z$  and  $s_Z \equiv \sin \theta_Z$ , as defined by

$$c_Z^2 s_Z^2 = \frac{e^2}{4\sqrt{2} G_F M_Z^2}, \quad (27)$$

rather than the leading order mixing angle  $\theta$ .

Let us begin with the coupling of the photon of the form  $(W^{+\mu\nu} W_\mu^- - W^{-\mu\nu} W_\mu^+) A_\nu$ . In terms of the wave-functions  $v_{\gamma,W}$ , this coupling is proportional to

$$g_\gamma = \sum_{i,j} g_i v_\gamma^i v_W^j \tilde{Z}_{ij} v_W^j. \quad (28)$$

From eqn. (14), we have  $g_i v_\gamma^i \equiv e$  and therefore, by applying the normalization condition  $\vec{v}_W^T \tilde{Z} \vec{v}_W = 1$ , we

<sup>2</sup> An alternative approach, Degenerate BESS [20, 21], produces degenerate vector and axial mesons and  $\alpha S = 0$  using a different theory without unitarity delay [10] – see “case I” described in [22].

obtain  $g_\gamma \equiv e$  independent of any choice of the four-site parameters — as required by gauge-invariance and consistent with the form of eqn. (26).

Next, we evaluate  $\Delta\kappa_\gamma$ , with

$$e[1 + \Delta\kappa_\gamma] = \sum_{i,j} g_i (v_W^i)^2 \tilde{Z}_{ij} v_\gamma^j = e \sum_{i,j} \frac{g_i}{g_j} (v_W^i)^2 \tilde{Z}_{ij}, \quad (29)$$

for which we calculate

$$\Delta\kappa_\gamma = \frac{\varepsilon f_1^4}{(f_1^2 + 2f_2^2)^2} x^2 = \frac{\varepsilon v^4}{f_2^4} x^2. \quad (30)$$

Note that this vanishes in the absence of wavefunction mixing ( $\varepsilon \rightarrow 0$ ), and also in the “three-site” limit ( $v/f_2 \rightarrow 0$ ), as consistent with [6].

Similarly we may compute  $\Delta g_1^Z$  and  $\Delta\kappa_Z$ , and we find

$$\begin{aligned} \Delta g_1^Z &= \Delta\kappa_Z + \frac{\varepsilon f_1^4 \tan^2 \theta_Z x^2}{(f_1^2 + 2f_2^2)^2}, \\ &= -\frac{(\varepsilon s_Z^2 f_1^4 + (1 + \varepsilon) f_1^2 f_2^2 + (1 + \varepsilon) f_2^4) x^2}{(f_1^2 + 2f_2^2)^2 \cos(2\theta_Z) c_Z^2}, \end{aligned} \quad (31)$$

where the difference between  $\theta$  and  $\theta_Z$  is irrelevant to this order. Note that  $\Delta g_1^Z - \Delta\kappa_Z$  vanishes when  $\varepsilon \rightarrow 0$ , and also, as expected [6], in the “three-site” limit  $f_2 \rightarrow \infty$ .

### B. $\rho, a_1 \rightarrow W + \gamma$

Finally, we consider the  $(\rho, a_1) - W - \gamma$  couplings that motivated this study. Electromagnetic gauge-invariance implies that the coupling of the form  $(\rho^{+\mu\nu} W_\mu^- - W^{-\mu\nu} \rho_\mu^+) A_\nu$  must vanish. Analogous to eqn. (28) we find that the  $\rho - W - \gamma$  and  $a_1 - W - \gamma$  couplings of this form are proportional to  $\vec{v}_W^T \tilde{Z} \vec{v}_{\rho, a_1} \equiv 0$ , and therefore vanish identically.

There is no reason, however, that terms proportional to  $(\rho_\mu^+, a_{1\mu}^+) W_\nu^- A^{\mu\nu}$  must vanish [5, 7]. In this case, we find

$$e \kappa_{\gamma W \rho} = \sum_{i,j} g_i v_W^i v_\rho^j \tilde{Z}_{ij} v_\gamma^j = e \sum_{i,j} \frac{g_i}{g_j} v_W^i v_\rho^j \tilde{Z}_{ij}, \quad (32)$$

and similarly for the  $a_1$ . Computing these couplings to

$\mathcal{O}(x^3)$ , we find

$$\kappa_{\gamma W \rho} = -\frac{\varepsilon(1 + \varepsilon)^{3/2} f_1^4}{2\sqrt{2}(f_1^2 + 2f_2^2)(\varepsilon f_1^2 + (1 + \varepsilon)f_2^2)} x^3 \quad (33)$$

$$\kappa_{\gamma W a_1} = \frac{\sqrt{2} \varepsilon v^2}{\sqrt{1 - \varepsilon} f_2^2} x. \quad (34)$$

Note that both couplings vanishes in the  $\varepsilon \rightarrow 0$  and  $f_2 \rightarrow \infty$  limits. Furthermore, while the  $\rho - W - \gamma$  coupling is typically small ( $\mathcal{O}(x^3)$ ), we find the  $a_1 - W - \gamma$  coupling is only suppressed by  $x$ , consistent with [5, 7]. If the value of  $\varepsilon$  corresponds (25) to  $\alpha S = 0$ , then  $\kappa_{\gamma W a_1}$  is

$$\kappa_{\gamma W a_1} = -\frac{2\sqrt{2}v^2(f_1^2 + f_2^2)x}{(f_1^2 + 2f_2^2)\sqrt{f_1^2 + 2f_1^2 f_2^2 + 2f_2^2}}. \quad (35)$$

As mentioned earlier, for this value of  $\varepsilon$ , the  $a_1$  state is lighter than the  $\rho$ .

## VI. SUMMARY

We have introduced a deconstructed Higgsless model with four sites and non-trivial wavefunction mixing, and have shown that it exhibits key features of holographic technicolor [5, 7]. The electroweak parameter  $\alpha S$  vanishes for a value of the wavefunction mixing at which the  $a_1$  is lighter than the  $\rho$  — even if all fermions are brane-localized. Furthermore, the model includes the decay  $a_1 \rightarrow W\gamma$ , suppressed by only one power of  $(M_W/M_\rho)$ , in contrast with an  $(M_W/M_\rho)^3$  suppression of the decay  $\rho \rightarrow W\gamma$ . These decays are of potential phenomenological interest at LHC.

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